P P SAVANI UNIVERSITY

Fourth Semester of B. Tech. Examination

May 2019

SESH2051 : Mathematical Methods for Computation

[Date:13/05/2019, Monday] **[Time:09:00 A.M. to 11.30 A.M.]** **[Total Marks:60]**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION-I

Q-1 Answer the following.(Any Five) **[05]**

(i) The order (O) and degree (D) of the ODE $\frac{d^2y}{dx^2} = x + \left(\frac{dy}{dx}\right)^{\frac{1}{2}}$

a) $O = 2,\ D = 1$ b) $O = 2,\ D = 2$ c) $O = 1,\ D = 1/2$ d) $O = 2,\ D = 1/2$

(ii) If $(ax + by + c)dx + (ex + fy + g)dy = 0$ is exact, then

a) $a = f$ b) $b = e$ c) $a = e$ d) $b = f$

(iii) Particular integral of ODE $(D - 1)^2 y = e^x$ is

a) $\frac{x^2}{2}e^x$ b) e^x c) $\frac{x^3}{6}e^x$ d) none of the above

(iv) $L\{e^{-t} + e^{t}\}$ is

a) $\frac{s}{s^2-1}$ b) $\frac{2s}{s^2-1}$ c) $\frac{2}{s^2-1}$ d) $\frac{s}{s-1}$

(v) $L^{-1}\{\frac{1}{s^3}\}$ is

a) t^2 b) $t/3$ c) $t^2/2$ d) $t^2/3!$

(vi) $L\{2^t\}$ is

a) $\frac{1}{s - \log 2}$ b) $\frac{2}{s^3}$ c) $\frac{1}{s-2}$ d) does not exist

(vii) If $L\{f(t)\} = F(s)$, then $L\{tf(t)\}$ is

a) $F(s-1)$ b) $e^{-s}F(s)$ c) $-F'(s)$ d) $-sF(s)$

Q-2 Answer the following. **[10]**

(a) Verify that the ODE $(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)dy = 0$ is exact and solve it.

(b) Solve using method of variation of parameter $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin x$

OR

(a) Solve the ODE $(D^2 + 3D + 2)y = e^{2x}\sin x$

(b) Solve using method of undetermined coefficients $\frac{d^2y}{dx^2} + y = 2\cos x$

Q-3 Answer the following. (Any Two) **[10]**

(a) Solve $z(px - qy) = y^2 - x^2$

(b) Solve $yp = 2xy + \log q$

(c) Solve the PDE $(D^2 - DD' - 6D'^2)z = \cos(2x + y)$

(d) Solve the PDE $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y}$

Q-4 Answer the following. (Any One) **[05]**

(a) Evaluate $L^{-1}\left\{\log\frac{s+2}{s+1}\right\}$

(b) Using Laplace transform show that $\int_0^\infty te^{-3t}\sin t\,dt = \frac{3}{50}$

SECTION-II

Q-1 Answer the following. (Any Five) **[05]**

(i) In Fourier series expansion, When $f(x)$ is even function

a) $a_0 = 0$ b) $b_0 = 0$ c) $a_n = 0$ d) $b_n = 0$

(ii) Interquartile range (IQR) =

a) $Q_2 - Q_1$ b) $\frac{1}{2}(Q_2 - Q_1)$ c) $\frac{1}{2}(Q_3 - Q_1)$ d) $Q_3 - Q_1$

(iii) Converting X to Standard Normal Variable Z

a) $Z = \frac{X+\mu}{\sigma}$ b) $Z = \frac{X-\sigma}{\mu}$ c) $Z = \frac{X+\sigma}{\mu}$ d) $Z = \frac{X-\mu}{\sigma}$

(iv) The mean of Binomial Probability distribution $P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}$

a) $\sqrt{np}$ b) $(np)^2$ c) $np/2$ d) np

(v) The probability that a standard normal variate $Z = 0$ is

a) 1 b) −1 c) 0 d) 0.5

(vi) Population Variance σ^2

a) $\frac{\sum(x_i - \mu)}{N-1}$ b) $\frac{\sum(x_i - \mu)}{N}$ c) $\frac{\sum(x_i - \mu)^2}{N-1}$ d) $\frac{\sum(x_i - \mu)^2}{N}$

Q-2 Answer the following. **[10]**

(a) Calculate Median from the distribution of marks obtained by 72 students.

Marks Group	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	24	14	9	8	9	5	3

(b) The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane

(i) arrives on time, given that it departed on time, and

(ii) departed on time, given that it has arrived on time.

(iii) arrives on time, given that it did not depart on time.

OR

(a) Find the half range sine series of $f(x) = e^{ax}$ in the interval $(0, \pi)$.

(b) Calculate the Karl Pearson's coefficient of correlation from the following data

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	37

Q-3 Answer the following. (Any Two) **[10]**

(a) Using Fourier integral representation, show that $\int_0^{\infty} \frac{\cos \omega x + \omega \sin \omega x}{1+\omega^2} d\omega = \begin{cases} 0, & x < 0 \\ \frac{\pi}{2}, & x = 0 \\ \pi e^{-x}, & x > 0 \end{cases}$

(b) Calculate deviations and squared deviations about the mean for the following class size sample data. Also find the sample variance, standard deviation and coefficient of Variation.

No of student	46	54	42	46	32

(c) The probability that a bomb will hit the target is 0.2. Two bombs are required to destroy the target. If six bombs are used, find the probability that the target will be destroyed.

Q-4 Answer the following. (Any One) **[05]**

(a) Calculate coefficient of correlation and obtain regression line of y on x for the following data and estimate y corresponding to $x = 6.2$

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(b) On average, 30-minute television sitcoms have 22 minutes of programming. Assume that the probability distribution for minutes of programming can be approximated by a uniform distribution from 18 minutes to 26 minutes.

(i) What is the probability that a sitcom will have 25 or more minutes of programming?

(ii) What is the probability that a sitcom will have between 21 and 25 minutes of programming?

(iii) What is the probability that a sitcom will have more than 10 minutes of commercials or other non programming interruptions?

* * *